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# Joint Inversion of Receiver Functions and Apparent Incidence Angles to Determine the Crustal Structure of Mars

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1           **Joint Inversion of receiver functions and apparent**  
2           **incidence angles to determine the crustal structure of**  
3           **Mars**

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12           **Key Points:**

- 13           • We apply recent results from random matrix theory to identify crustal phases in noisy  
14           receiver functions for Mars from InSight data
- 15           • Once identified, we jointly invert these phases with frequency-dependent apparent  
16           S-wave velocity curves
- 17           • Results show a crustal thickness of 43 km with two inter-crustal discontinuities at 8  
18           km and 21 km beneath the lander

## Abstract

Recent estimates of the crustal thickness of Mars show a bimodal result of either  $\sim 20$  km or  $\sim 40$  km beneath the InSight lander. We propose an approach based on random matrix theory applied to receiver functions to further constrain the subsurface structure. Assuming a spiked covariance model for our data, we first use the phase transition properties of the singular value spectrum of random matrices to detect coherent arrivals in the waveforms. Examples from terrestrial data show how the method works in different scenarios. We identify three previously undetected converted arrivals in the InSight data, including the first multiple from a deeper third interface. We then use this information to jointly invert receiver functions with the absolute S-wave velocity information in the polarization of body waves. Results show a crustal thickness of  $43 \pm 5$  km beneath the lander with two mid-crustal interfaces at depths of  $8 \pm 1$  km and  $21 \pm 3$  km.

## Plain Language Summary

Recent analysis of seismic data from InSight shows that the crustal thickness beneath the InSight lander can be either 20 km or 40 km. To resolve this ambiguity, we apply results from random matrix theory to receiver function analysis. The distribution of singular values of a random matrix shows well-behaved deterministic properties that can be used to separate them from those of an underlying coherent signal if present. We use examples from terrestrial data to show how the method works. When applied to receiver functions computed from InSight seismic data, we identify three new energy arrivals, including one that supports the existence of a deeper third layer. Using this information, we simultaneously inverted the receiver function data along with the measured incidence angle of body waves. Results show a crustal thickness of  $43 \pm 5$  km beneath the lander with two mid-crustal interfaces at depths of  $8 \pm 1$  km and  $21 \pm 3$  km.

## 1 Introduction

The InSight mission landed in the Elysium Planitia plain of Mars on November 26, 2018 (Banerdt et al., 2020) and deployed a three component very broadband seismometer (SEIS) (Lognonné et al., 2019, 2020) on the surface. Along with measuring the seismicity and the present thermal state of its interior, a primary goal of the mission is to constrain the interior structure of Mars. In comparison with the Earth, Mars has a low seismicity rate with quakes of smaller magnitude ( $2-5 M_w$ ) (Giardini et al., 2020). Receiver function (RF) analysis is a robust single station technique that can be used in this case to constrain the crustal structure. Primary body waves (P and S) give rise to converted secondary phases (Ps and Sp) when they impinge upon a seismic discontinuity from beneath. RFs exploit these converted phases to gain information about the discontinuities in the crust and upper mantle. They have previously been used to investigate the thickness of the lunar crust using seismic data from the Apollo missions (Vinnik et al., 2001; Lognonné et al., 2003; Gagnepain-Beyneix et al., 2006). Using the data from the InSight mission, Lognonné et al. (2020) computed RFs from two marsquakes and showed evidence of subsurface layering with low seismic velocities in the first upper 8–11 km. Recently, Knapmeyer-Endrun et al. (2021) used RFs from three marsquakes and showed the observations to be consistent with either a two-layer model with the Moho at  $20 \pm 5$  km or a three-layer model with the Moho at  $39 \pm 8$  km depth below the lander. Although the thicker model is more compatible with geodynamical constraints, this ambiguity could not be resolved from the data due to a lack of phase move-out information and excessive noise in the later part ( $> 10$  s) of the waveforms which inhibited the identification of multiple arrivals. Compaire et al. (2021) and Schimmel et al. (2021) analyzed ambient field autocorrelations and identified reflection signals consistent with the first two interfaces. Li et al. (2022) confirmed the first interface at  $\sim 8$  km depth and the anisotropic nature of the layer above based on SH-wave reflections. Kim et al. (2021) and Durán et al. (2022) later used updated RF datasets with

69 more events to provide additional constraints and connoted a preference for the three-layer  
 70 crustal model. Khan et al. (2021) and Drilleau et al. (2022) arrived at similar results using  
 71 body wave travel-times. In this paper, we build upon the previous work of Knapmeyer-  
 72 Endrun et al. (2021) to infer further constraints on the crustal structure of Mars using new  
 73 techniques and additional data from the InSight mission.

74 We first focus on the problem of detection of multiple phases in our selected RF dataset.  
 75 For this, we propose a method that utilises recent results from the random matrix theory  
 76 to extract coherent signals in the RF waveforms. Assuming the observed signal to be a  
 77 superposition of random noise and an underlying low-rank signal, the eigenvalues of the  
 78 data covariance matrix follow a well behaved and deterministic limiting spectral distribu-  
 79 tion dictated by the generalized Marchenko-Pastur law. This information can be effectively  
 80 used to decouple and identify coherent signal eigenvalues reflecting primary subsurface fea-  
 81 tures from a bulk spectrum formed by incoherent scattering, random noise, and small-scale  
 82 heterogeneity with distinct eigenvector rotation properties. Once identified, the secondary  
 83 phase arrivals together with the primary conversions from crustal interfaces can be used to  
 84 invert for the structure. We then address the problem of non-uniqueness of RF inversions.  
 85 Being primarily sensitive to shear velocity contrasts of interfaces and relative travel-time  
 86 of converted waves, inversions of RF data alone can be affected by depth velocity trade-off  
 87 (Ammon, 1991). They are therefore usually inverted jointly with other independent data  
 88 sets that provide additional constraints on absolute shear wave velocities like surface-wave  
 89 dispersion (e.g. Du & Foulger (1999); Julia et al. (2000); Bodin et al. (2012)). Svenningsen  
 90 & Jacobsen (2007) showed that P-wave polarization can also be used to constrain the S-wave  
 91 velocity structure of the subsurface using a simple relation between the observed apparent  
 92 incidence angle and half-space S-wave velocity (Wiechert, 1907). Following this, we previ-  
 93 ously showed how a joint inversion of apparent velocity curves and receiver function data  
 94 can lead to a well constrained velocity structure for limited data sets comprising only a few  
 95 events (Joshi et al., 2021). We adopt a similar methodology here to jointly invert an RF  
 96 dataset with a mean apparent velocity curve using a transdimensional approach.

## 97 **2 Data and Method**

### 98 **2.1 RF processing**

99 InSight has identified 1244 marsquakes (InSight Marsquake Service, 2022) since its  
 100 operations started in 2018. Each quake is assigned a type and quality depending on its  
 101 energy content and uncertainty in location estimate (Giardini et al., 2020). Only a few  
 102 of these marsquakes generate waves that propagate through the mantle like teleseismic  
 103 earthquakes, most of which do not have a precise location. Our database for Mars thus  
 104 consists of 8 LF and BB seismic events (InSight Mars SEIS Data Service, 2019) with high  
 105 SNR and event quality A-B (J. F. Clinton et al., 2021). Most of these events have similar  
 106 distances and back-azimuths as they all originate in the Cerberus Fosse region which is a  
 107 young tectonic structure located to the east of the lander. S0183a is located farther away  
 108 but we nevertheless use it as its inclusion does not have a significant effect on the results.  
 109 For the terrestrial example, we use data from seismic station VSU in Vasula, Estonia. We  
 110 select events with a similar back-azimuth and distance range to mimic the InSight data.  
 111 Details of the events used in this study are provided in the Supplementary Material (Tables  
 112 S1, S3, and S4).

113 To calculate RFs, we apply a time-domain Wiener filter for deconvolution as described  
 114 by Hannemann et al. (2017). We first remove the transfer functions from the individual  
 115 components of the data, rotate to ZNE coordinates as VBB uses the U, V, W component  
 116 system, and filter the seismograms between the corner frequencies (Table S1 in Supplemen-  
 117 tary Material) using a zero-phase Butterworth filter. Subsequently, the ZNE coordinate  
 118 system is rotated into ZRT to obtain radial and transverse components using the back-  
 119 azimuth estimates provided by the Marsquake Service (MQS) (J. Clinton et al., 2018). For

120 S0784, a back-azimuth of  $100^\circ$  was determined by comparing RFs across different azimuths.  
 121 A Wiener filter is determined such that it transforms the P-wave signal on the vertical  
 122 component into a band-limited spike. All the components of the data are then folded with  
 123 this filter to obtain the RFs. The terrestrial data was processed similarly but was filtered  
 124 between 5 Hz and 50 s.

## 125 2.2 Phase identification

126 In RF data, the travel-times of the converted phases relative to the direct P arrival  
 127 depend on the epicentral distance. This is generally seen as phase move-out which is dif-  
 128 ferent for direct and multiple phases, and helps to distinguish between these. For complex  
 129 structures with dipping interfaces and seismic anisotropy, the travel-times and amplitudes of  
 130 conversions also vary with back-azimuth. Although events generated from similar epicentral  
 131 distance and back-azimuths should theoretically have coherent conversions and multiples,  
 132 interference with the scattered wave-field, small-scale heterogeneity, and random noise gener-  
 133 ates variations which can be seen as perturbations superimposed on the response of the  
 134 primary sub-surface feature. The observed RF data matrix,  $Y_{n \times m} = X_{n \times m} + \sigma Z_{n \times m}$ , can  
 135 now be modelled as a fixed rank perturbation ( $\text{rank}[X] = r \leq n$ ) of the random noise  
 136 matrix  $Z \sim \mathcal{N}(0, 1)$ . This is known as the spiked covariance model (Johnstone, 2001). To  
 137 extract an approximation of the uncontaminated response  $\hat{X}(Y)_{n \times m} \approx X_{n \times m}$ , we exploit  
 138 the fact that the asymptotic eigenvalue distribution of the covariance of a random matrix  
 139 follows the Marchenko-Pastur (MP) law (Marchenko & Pastur, 1967) which has a compact  
 140 support  $\Omega$  with bounds  $\lambda_{\pm}$

$$\Phi(\lambda|\sigma, \gamma) = \begin{cases} \frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{2\pi\lambda\gamma\sigma}, & \lambda_- \leq \lambda \leq \lambda_+ \\ 0, & \text{otherwise} \end{cases}, \text{ with } \lambda_{\pm} = \sigma^2(1 \pm \sqrt{\gamma})^2 \quad (1)$$

141  $\Phi$  denotes the probability density of eigenvalues.  $\lambda_-$ ,  $\lambda_+$ ,  $\sigma$ , and  $\gamma$  denote the smallest  
 142 eigenvalue, largest eigenvalue, noise level and matrix aspect ratio  $n/m$ , respectively.  $\lambda_{\pm}$  fluctu-  
 143 ate on the small scale  $n^{-2/3}$  according to the Tracy-Widom distribution (Tracy & Widom,  
 144 1996). Qualitatively, the empirical distribution of the eigenvalues of  $Z$  forms a deformed  
 145 quarter circle bulk with bulk edges given by  $\lambda_{\pm}$  and bulk width  $4\sqrt{\gamma}\sigma^2$ , and all eigenvalues  
 146 lie strictly within these bounds. The eigenvalues show a sort of self-arranging behaviour  
 147 which, in presence of a non-random sample coherency (i.e.,  $X \neq 0$ ), have a repulsion effect  
 148 on the signal eigenvalue if present. Thus we see a phase transition phenomenon (Baik et al.,  
 149 2005) where, above a certain signal threshold, the signal eigenvalues separate away from the  
 150 bulk "noise" eigenvalues and converge asymptotically to a different distribution. The same  
 151 follows for the singular values which scale as the square root of the eigenvalues (Benaych-  
 152 Georges & Nadakuditi, 2012). Setting  $X_n = \sum_{i=1}^m a_{n,i}x_i b_{n,i}^T$  and  $Y_n = \sum_{i=1}^m u_{n,i}y_i v_{n,i}^T$ , the  
 153 BBP (Baik–Ben Arous–Péché) phase transition results in a mapping of singular values  $y_i$   
 154 of the observed matrix  $Y$  to  $x_i$  of the uncontaminated low rank signal  $X$ :

$$y_i \xrightarrow{\text{a.s.}} \begin{cases} \sigma(1 + \sqrt{\gamma}) & x_i \leq \sigma\gamma^{\frac{1}{4}} \\ \sqrt{(x_i + \frac{\sigma}{x_i})(x_i + \gamma\frac{\sigma}{x_i})}, & x_i > \sigma\gamma^{\frac{1}{4}} \end{cases} \quad (2)$$

155 Similarly, the left and right singular vectors pairs  $(u_i, a_i)$  and  $(v_i, b_i)$  are orthogonal  
 156 within the bulk but become strongly correlated and show a non-zero dot product past the  
 157 critical point:

$$|\langle a_{n,i}, u_{n,j} \rangle|^2 \xrightarrow{\text{a.s.}} \begin{cases} \frac{x_i^4 - \gamma}{x_i^4 + \gamma x_i^2}, & x_i = x_j \\ 0, & x_i \neq x_j \end{cases} \quad (3)$$

$$|\langle b_{n,i}, v_{n,j} \rangle|^2 \xrightarrow{\text{a.s.}} \begin{cases} \frac{x_i^4 - \gamma}{x_i^4 + x_i^2}, & x_i = x_j \\ 0, & x_i \neq x_j \end{cases} \quad (4)$$

Using these transition equations, Gavish & Donoho (2014) derive an analytical expression for the optimal rank- $r$  approximation of the data matrix  $\hat{X}(Y)$  by minimising the asymptotic mean squared error of their misfit  $\|X - \hat{X}(Y)\|_F^2$  over all singular values  $u_i > \sigma\sqrt{1+\gamma}$  and  $0 < \gamma \leq 1$ . For the complete derivation, see Gavish & Donoho (2014). This results in an expression for a threshold value  $\tau$

$$\tau = \lambda_*(\gamma) \cdot \sqrt{n}\sigma \quad (5)$$

where

$$\lambda_* = \sqrt{(2\gamma + 1) + \frac{8\gamma}{(\gamma + 1) + \sqrt{(\gamma^2 + 14\gamma + 1)}}} \quad (6)$$

This threshold marks the unique transition point of the signal singular values from those of random noise matrix  $Z$  for a given spectral distribution of  $Y$  with noise  $\sigma$ , taking into account the support fluctuations. The median of a standard MP distribution ( $\sigma = 1$ ) is given by

$$\mu_{MP} = \frac{1}{2\pi t} \int_{\lambda_-}^x \sqrt{(\lambda_+ - t)(t - \lambda_-)} dt \quad (7)$$

158 The noise  $\sigma$  can be estimated by matching the median of the standard MP distribution  
159 to that of the bulk singular values. This results in a robust noise estimator that estimates  
160 noise by comparing the perturbed singular values with the MP distribution

$$\hat{\sigma}(Y) := \frac{\lambda_{med}}{\sqrt{n\mu_{MP}}} \quad (8)$$

161 Using  $\hat{\sigma}(Y)$  for  $\sigma$  in eq. 5, we get

$$\hat{\tau}_* = \omega(\gamma) \cdot \lambda_{med} \quad (9)$$

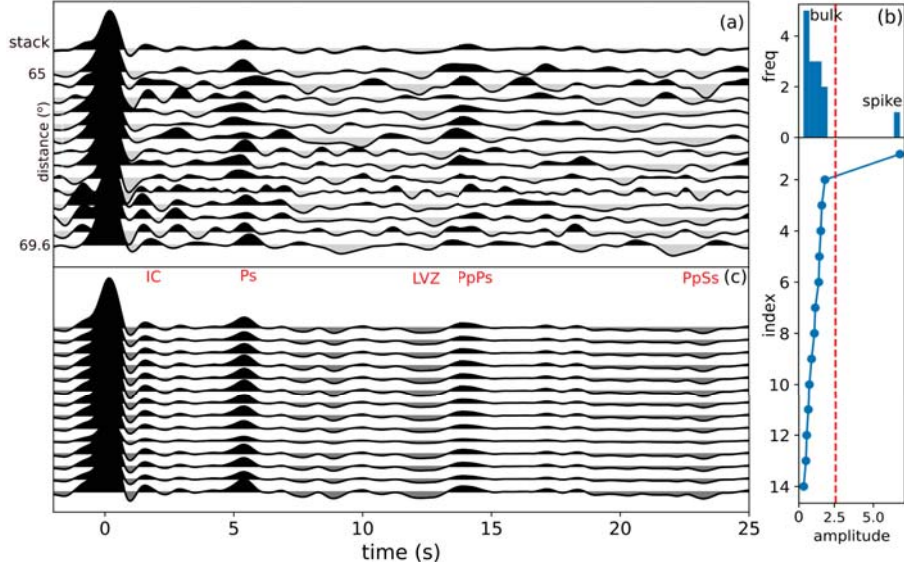
where

$$\omega(\gamma) \approx 0.56\gamma^3 - 0.95\gamma^2 + 1.82\gamma + 1.43 \quad (10)$$

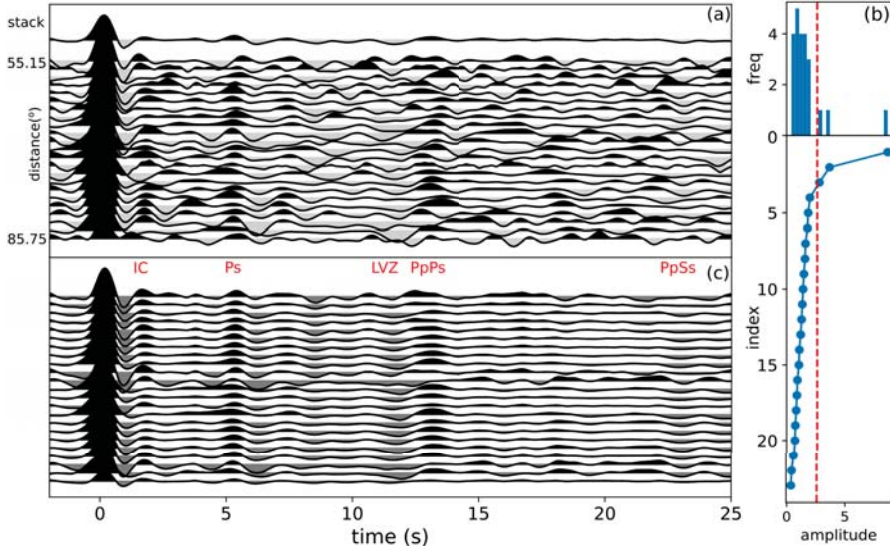
162 In the final step, the singular value matrix  $y_i$  is replaced by  $\hat{y}_i$  where the values below  
163 the threshold limit given by eq. 5 are padded and the approximate uncontaminated signal  
164 matrix  $\hat{X}(Y)$  is reconstructed from the observations  $Y$  using  $\hat{X}(Y) = \sum_{i=1}^m u_{n,i} \hat{y}_i v_{n,i}^T$ . A  
165 sample algorithm is provided in the Supplementary Material. In essence, the outlined proce-  
166 dure provides objective criteria to select the number of principal components that describe  
167 the variance of the structural signal by modelling noise as an additive independent and  
168 identically distributed (i.i.d.) random matrix.

### 169 2.3 Apparent velocity curves

In order to reduce non-uniqueness in the inversion of receiver functions, we use the absolute S-wave velocity information contained within the P-wave polarization as a complementary constraint. A consequence of Snell's law, the relation between the apparent P-wave incidence angle ( $\bar{i}_p$ ) and the subsurface S-wave velocity was derived by Wiechert (1907) and is given by eq. 10. Here  $p$  denotes the ray parameter. This apparent P-wave incidence angle can be calculated directly from the amplitudes of vertical and radial receiver functions at



**Figure 1.** (a) Raw RFs from terrestrial station VSU in epicentral distance range  $65^\circ - 69^\circ$  and back-azimuth  $10^\circ - 40^\circ$  (b) distribution of the singular spectrum (top) and the singular values arranged in decreasing amplitude (below). The red dashed line denoted the noise threshold. (c) RFs reconstructed using singular values above the noise threshold. The marked phases represent the Ps phases of an intracrustal discontinuity (IC), the Moho (Ps) and a low-velocity zone (LVZ), and the Moho PpPs and PsPs+PpPs phases, respectively.



**Figure 2.** Same as Figure 1 but for epicentral distance range  $55^\circ - 85^\circ$  and back-azimuth between  $80^\circ - 120^\circ$ .

time  $t=0$ , as described in Svenningsen & Jacobsen (2007)

$$v_{S,app} = \sin\left(0.5\bar{v}_p\right)/p \quad (11)$$

$$\tan \bar{i}_p = \frac{RRF(t=0)}{ZRF(t=0)} \quad (12)$$

170 Following a similar procedure as Knapmeyer-Endrun et al. (2018) and estimating  $i_p$  as a  
 171 function of low pass Butterworth filter period ( $T$ ), we calculate a frequency-dependent S-  
 172 wave velocity curve  $v_S(T)$  which emphasises the absolute S-wave velocity variation with  
 173 depth. A mean RF is calculated from all the raw RF waveforms in the dataset. This is then  
 174 used to compute a  $v_{S,app}$  curve which is jointly inverted together with the mean RF. We  
 175 measure the dominant period of the spike in the mean ZRF and discard the values of filter  
 176 periods smaller than that.

## 177 2.4 Inversion

178 A Markov-chain Monte Carlo (MCMC) transdimensional Bayesian inversion method  
 179 based on Bodin et al. (2012) (Dreiling & Tilmann, 2019) was used for the joint inversion of  
 180 the mean RF and  $v_{S,app}$  curve. In this formulation, the number of layers itself becomes an  
 181 unknown and is also inverted for along with the other model parameters. The solution is an  
 182 ensemble of models that are distributed according to a posterior probability density function  
 183 given by Bayes's rule. Each layer is parameterised by depth,  $V_s$  and the  $v_P/v_S$  ratio. Density  
 184 is not inverted for but is calculated using Birch's law (Birch, 1961). We use flat model priors  
 185 and their ranges for depth,  $V_s$  and  $v_P/v_S$  ratio were set to 0 – 100 km, 1 – 5 km/s and  
 186 1.4 – 2.2, respectively. A maximum of 20 layers was imposed and the range for the noise  
 187 amplitude was set to 0.01 – 0.5 with correlation values of RF and  $v_{S,app}$  data fixed to 0.96  
 188 and 0, respectively. For calculating synthetic RFs, we use the forward calculation module  
 189 implemented by Shibutani et al. (1996). The algorithm calculates the impulse response of  
 190 a layer stack in the P-SV system. The resulting synthetic Z- and RRFs are convolved with  
 191 the observed ZRF in order to account for the observed waveform complexity (Knapmeyer-  
 192 Endrun et al., 2018). A  $v_{S,app}$  profile is then calculated for the RFs using the procedure  
 193 described in Section 2.3.

## 194 3 Results

195 To illustrate the method, we show its application on data from the terrestrial seismic  
 196 station VSU. Figure 1(a) shows the raw data which consists of RFs computed from closely  
 197 located events. In general, the data are noisy. Subplot (b) shows the distribution of singular  
 198 values and its spectrum for the data. We see the general singular value repulsion behaviour  
 199 with a "bulk" noise region well separated away from the signal "spike". This noise bulk  
 200 follows the limiting spectral distribution given by the MP law with extreme eigenvalues and  
 201 their variance given by Eq. 1 and 8. The red line shows the optimal threshold for singular  
 202 value truncation when noise is modelled as an independent and identically distributed (i.i.d.)  
 203 random matrix. Using the singular value above this threshold, we reconstruct the data by  
 204 projecting it onto the corresponding eigenvector. Subplot (c) shows the reconstructed RF  
 205 data showing clear coherent energy arrivals at  $\sim 1$  s, 5 s, 12 s, 14 s and 23 s. We interpret  
 206 these as the Ps phases of an intracrustal discontinuity (IC), the Moho (Ps) and a low-velocity  
 207 zone (LVZ), and the Moho PpPs and PsPs+PpSs phases, respectively. The reference timing  
 208 for the Moho Ps phase here is taken from Knapmeyer-Endrun et al. (2014). In general, the  
 209 method is applicable equally well to data sets covering a wider range of distances and back-  
 210 azimuths. The number of singular values above the threshold then increases to accommodate  
 211 the data variance. Figure 2 shows the reconstruction of RFs from similar back-azimuths  
 212 but a wider epicentral distance range ( $55^\circ - 85^\circ$ ). Here the threshold increases to three  
 213 to accommodate the move-out of various phases. Synthetic examples are provided in the  
 214 Supplementary Material (Fig. S1, S2)

215 Figure 3 shows the result of applying the phase identification methodology to our se-  
 216 lected data from the InSight mission. Apart from the three primary phases at 2.4 s, 4.8 s,

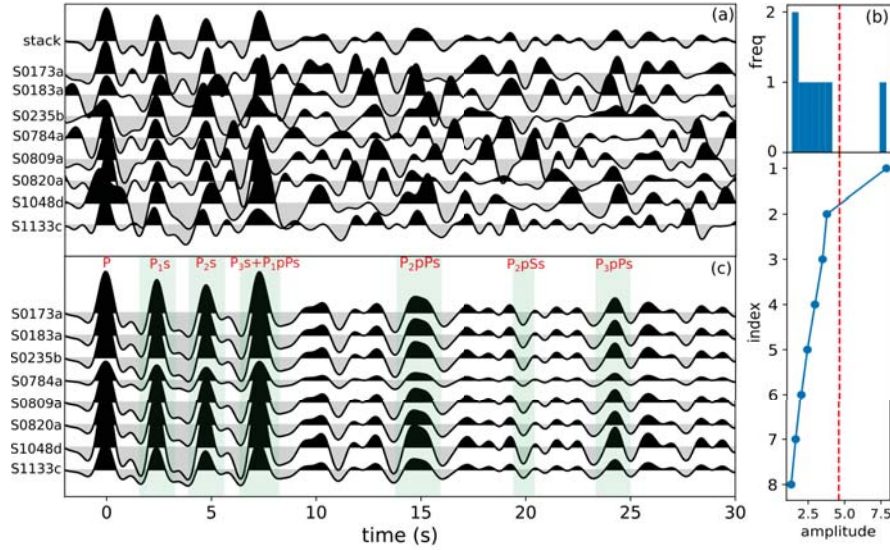


217 and 7.2 s previously identified in Knapmeyer-Endrun et al. (2021), the raw RF data (sub-  
 218 plot(a)) does not seem to contain any consistent phases after the initial 8 s. From subplot  
 219 (b), we see that the first principal component is sufficient to identify the main phase arrivals  
 220 within the first 30s of the RF waveforms. This is expected as all the events considered here  
 221 have similar distances and back-azimuths. The reconstructed RF waveforms are shown in  
 222 subplot (c). In addition to the three primary phases, we report three new multiple phases  
 223 at 15 s, 20 s, and 23 s. We interpret these as the P<sub>2</sub>pPs, P<sub>2</sub>pSs and P<sub>3</sub>pPs phases, where  
 224 the sub-scripted numeral in the phase name denotes the generating interface. Note that  
 225 the P<sub>3</sub>pPs phase holds significant importance as it corroborates the existence of the much  
 226 speculated third inter-crustal layer below the InSight landing site. These arrival timings are  
 227 used to define a misfit window for the RFs which are then jointly inverted with the mean  
 228  $v_{S,app}$  profile. We initialised 72 chains of 1,000,000 iterations, each sampling the model space  
 229 simultaneously and independently, with 500,000 iterations discarded as the burn-in phase.  
 230 Outlier chains were removed, and the models were thinned to obtain a final ensemble of  
 231 100,000 models. The main results of the inversion are shown in Figure 4.

232 A three-layer model exhibits the highest probability density in the solution ensemble.  
 233 Subplot (a) shows the posterior distribution of the  $v_S$  profiles as a function of depth, along  
 234 with the probability for each interface depth. We see two well-defined mid-crustal interfaces  
 235 at depths of  $8 \pm 1$  km and  $21 \pm 3$  km, along with the crust-mantle transition at  $43 \pm 5$  km.  
 236 The resulting crustal models agree well with the three-layer models presented in Knapmeyer-  
 237 Endrun et al. (2021) and Durán et al. (2022). The modelled  $v_{S,app}$  curves and the RFs follow  
 238 the observed data closely and are shown in subplots (b) and (c), respectively. Due to the  
 239 low SNR of the individual RF waveforms at longer periods, the  $v_{S,app}$  curves are limited  
 240 to periods  $< 13$  s. This helps provide tight constraints on the observed  $v_S$  value within  
 241 the shallow part of the crust, but the uncertainty increases with depth where the  $v_S$  values  
 242 and their increases are primarily controlled by the RF amplitudes. The estimated mean  
 243  $v_P/v_S$  ratio for the three layers is 1.82, 1.77 and 1.64, yielding an average value of  $\sim 1.75$   
 244 for the crust. A distinct negative arrival of unclear origin is seen at 11.5 s. Although we do  
 245 not include this in our analysis to avoid over-interpretation, it could either be a Pp phase  
 246 from the second interface or a low-velocity zone at a depth of  $\sim 70 - 75$  km. The P<sub>1</sub>pSs  
 247 would arrive earlier, between 8 – 10 s, ruling out this possibility. When combined with  
 248 the observed gravity field using results from Wicczorek et al. (2022), our crustal thickness  
 249 estimates predict a global average crustal thickness of 46 – 73 km, assuming a uniform  
 250 density crust of  $2600 \text{ kgm}^{-3}$  for Mars. If a higher crustal density is considered for the  
 251 northern lowlands, the global average crustal thickness of Mars will lie in the range of  
 252 34 – 73 km.

## 253 4 Discussion

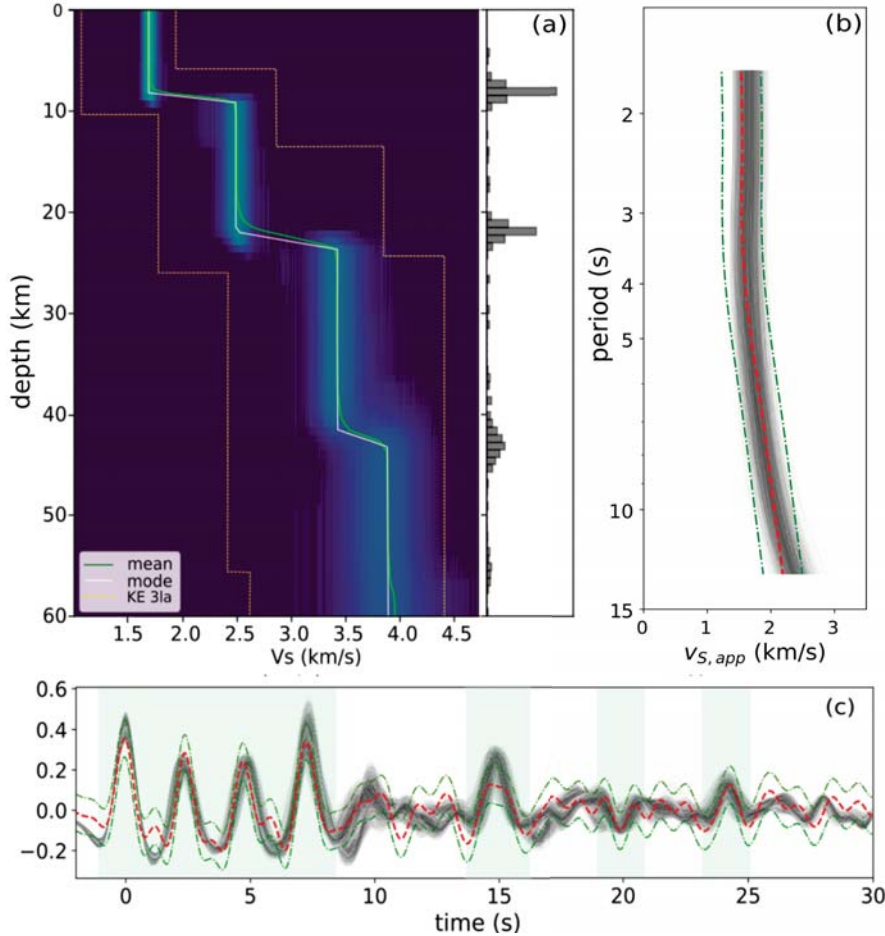
254 With just a handful of good quality, small magnitude and closely located marsquakes,  
 255 the analysis of the RF data from the InSight mission present us with many challenges. In  
 256 this work, we attempt to use this close distance range to our advantage to uncover addi-  
 257 tional features in the data using concepts of random matrix theory and principal component  
 258 analysis. For events from similar distances and back-azimuths, considerably fewer principal  
 259 components can reconstruct the main features present in the data. Additional components  
 260 are needed as distance and back-azimuth ranges increase. Using synthetics and real data,  
 261 Zhang et al. (2019) demonstrated that just the first few principal components could effec-  
 262 tively reconstruct all the data variance within events from varying back-azimuths. Here  
 263 we used events with varying distances and similar back-azimuths to establish an equivalent  
 264 idea. A few points are, however, worth noting. Occasionally, data reconstruction with a few  
 265 principal components can lead to an erroneous broadening of phases. Though it does not  
 266 affect the detection of phases, it could sometimes lead to an unwanted merge of very close  
 267 arrivals. We also find that the singular spectrum, and hence the resulting threshold, can  
 268 show slight variations based on the dataset’s quality. For highly irregular RF waveforms,



**Figure 3.** Same as Figure 1 but for Mars. The shaded regions show the denoted arrivals.

269 this could severely bias the threshold estimate to lower values. In this case, utilising higher  
 270 principal components for data reconstruction will likely result in individual waveform vari-  
 271 ations instead of emerging features like phase move-out and back-azimuth variations. The  
 272 compact support of the random singular values can sometimes be disconnected, and there-  
 273 fore, choosing the threshold based only visually on the histograms can lead to errors. On  
 274 the other hand, histogram bins might not always clearly reflect the transition gap from  
 275 random to signal singular values. A full computation of the threshold is therefore required.  
 276 The number of events is also an essential factor. As the size of the dataset ( $m, n$ ) increases,  
 277 the fluctuations of the Tracy-Widom distribution decrease. Thus, the larger the dataset,  
 278 the higher the stability of the threshold. For a small dataset, the assumption of the spiked  
 279 covariance model can break down. Finally, the noise in receiver functions is not entirely  
 280 random and generally has a finite covariance. A recent study by Donoho et al. (2020) gen-  
 281 eralises the spiked covariance model to include correlated noise. Extending this analysis to  
 282 include the correlated case is essential and will be the subject of a later study.

283 Various interpretations are available for the first two interfaces, ranging from a change  
 284 in porosity to chemical composition. The low seismic velocity of the first layer has been  
 285 attributed to a combination of high porosity (20 – 30%) and low-density lithology of the  
 286 region due to the presence of cements and aqueous alterations of minerals (Li et al., 2022b).  
 287 With increasing depth, material compaction and viscous deformation of host rock can lead to  
 288 the closure of pore spaces resulting in a transition zone between the porous and non-porous  
 289 material (Gyalay et al., 2020). Fractured ejecta deposits from the Utopia basin, change  
 290 in rock crystallinity, and the Borealis impact melt could also explain these crustal layering  
 291 (Wieczorek et al., 2022). Recent studies on surface wave dispersion show an almost constant  
 292 S-wave velocity of  $\sim 3.2$  km/s in the top 5–30 km depth range for the crustal structure away  
 293 from the InSight landing site (Kim et al., 2022). These values likely represent the average  
 294 crustal structure in the northern hemisphere, indicating that the topmost low-velocity layer  
 295 beneath the lander is plausibly a local feature.



**Figure 4.** (a) Posterior density of resulting  $V_s$  profiles and with histograms for interface depth. KE 3la represents the  $2\sigma$  bounds of the three-layer ensemble from Knappmeyer-Endrun et al. (2021). (b) Fit to the mean  $v_{S,app}$  curve (c) Fit to the mean RF waveform. The red dashed lines denotes the observed data and green dash-dotted lines represent the  $2\sigma$  uncertainty.

## 5 Summary and Conclusion

The receiver function method has played an important role in the analysis of the Martian crustal structure using data from the InSight mission. In order to contribute to that effort, here we present a method to identify coherent phase arrivals in noisy RF waveforms by modelling data noise as samples from an independent and identically distributed random matrix and using this information to jointly invert the RFs with apparent velocity curves. With examples from terrestrial data, we first show how only a few singular values can help reconstruct coherent parts of the signals enabling the detection of phase arrivals in RF waveforms. The number of singular values needed for this depends on the range of the distance and back-azimuths of the events in the dataset, with often a single value being sufficient in the special case of closely located events. We then apply this method to a set of 8 marsquakes detected by the InSight mission and evaluate the crustal structure below the landing site based on these data. Three new crustal phases were identified in the RF waveforms, which we interpret as  $P_2pPs$ ,  $P_2pSs$  and  $P_3pPs$  phases. A subsequent joint inversion of the RFs with the mean  $v_{S,app}$  curve shows that the crust of Mars below the

311 InSight landing site is composed of three distinct layers with increasing velocity. A crustal  
312 thickness of  $\sim 43$  km is estimated.

313 In conclusion, the results presented here agree well with previous work from Knapmeyer-  
314 Endrun et al. (2021). The identified P<sub>3</sub>PPs phase suggests a strong preference for the three-  
315 layer model in their study. The inversion results presented here have further constrained  
316 the subsurface velocities at the InSight landing site. Our preferred interpretation of the  
317 observed crustal layering beneath InSight considers this a result of a series of transitions from  
318 sediments or pyroclastic deposits that experienced aqueous alterations to less porous Utopia  
319 ejecta and finally to the pre-existing crustal materials from early differentiation of Borealis  
320 impact melt (Wieczorek et al., 2022). A thicker crust, like one obtained from such a three-  
321 layer model, is also more compatible with the amount of heat-producing elements within the  
322 Martian crust estimated by spectroscopy observations and geodynamical modelling (Taylor,  
323 2013; Knapmeyer-Endrun et al., 2021). A much lower bulk crustal density and significant  
324 enrichment in crustal heat-producing elements would otherwise be needed for a thinner crust  
325 (Knapmeyer-Endrun et al., 2021). However, it is unlikely that this three-layered formation  
326 is indicative of the global Martian crustal structure (Kim et al., 2022) and can be plausibly  
327 just a feature of the local geology in the vicinity of the InSight landing site.

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334 Contribution Number 188.

### 335 Open Research

336 Seismic data for station VSU are publically available and can be obtained from EIDA  
337 (<http://eida.gfz-potsdam.de/webdc3/>) using the event information provided in the sup-  
338 plementary material. The InSight seismic data presented here ([http://dx.doi.org/10](http://dx.doi.org/10.18715/SEIS.INSIGHT.XB-2016)  
339 [.18715/SEIS.INSIGHT.XB-2016](http://dx.doi.org/10.18715/SEIS.INSIGHT.XB-2016)) are publicly available through the Planetary Data System  
340 (PDS) Geosciences node of the Incorporated Research Institutions for Seismology (IRIS)  
341 Data Management Center under network code XB ([https://pds-geosciences.wustl.edu/](https://pds-geosciences.wustl.edu/missions/insight/seis.htm)  
342 [missions/insight/seis.htm](https://pds-geosciences.wustl.edu/missions/insight/seis.htm)), and through the data center of Institut de Physique du  
343 Globe, Paris (<http://www.seis-insight.eu>).

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Figure 1.



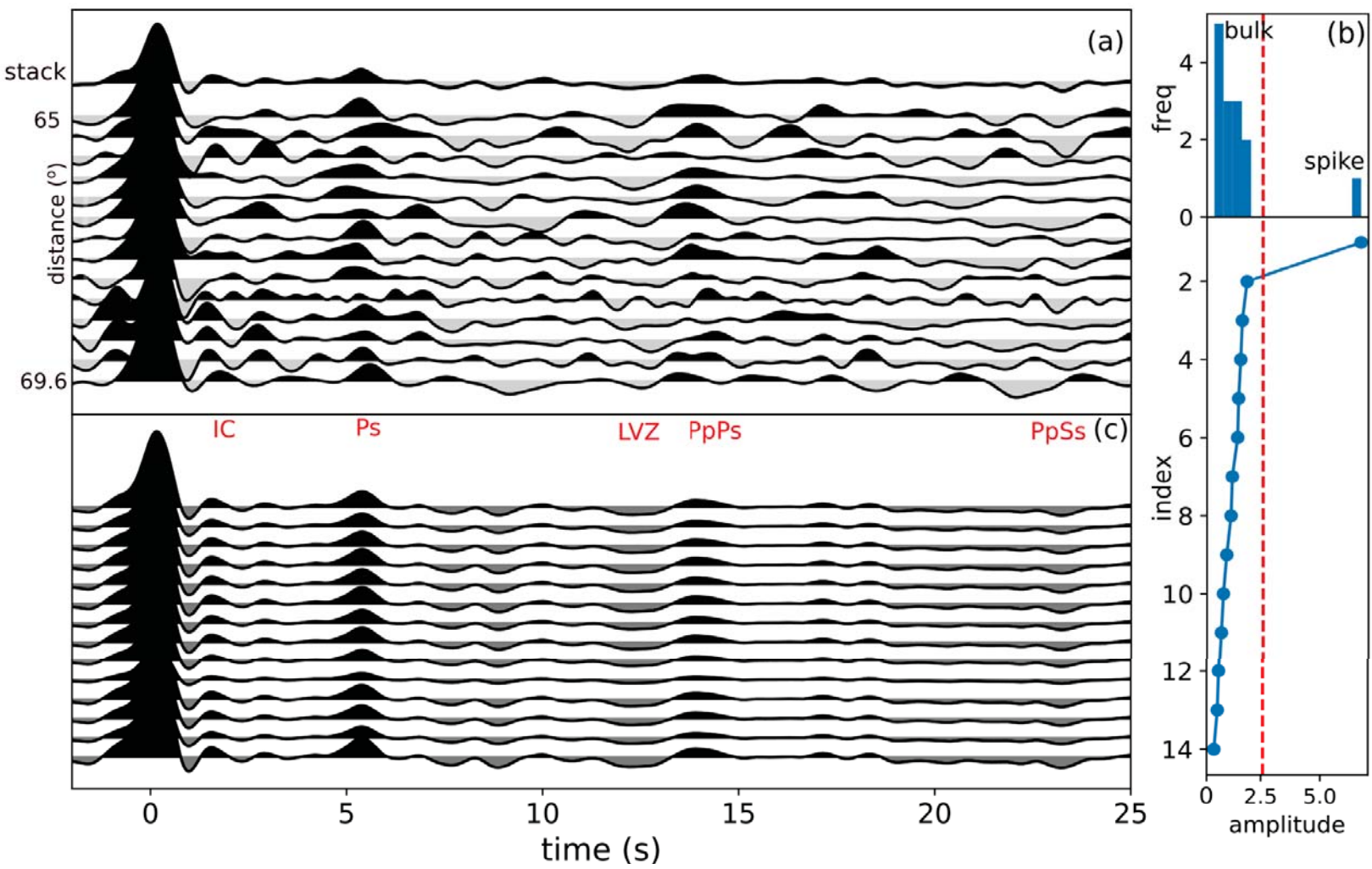


Figure 2.

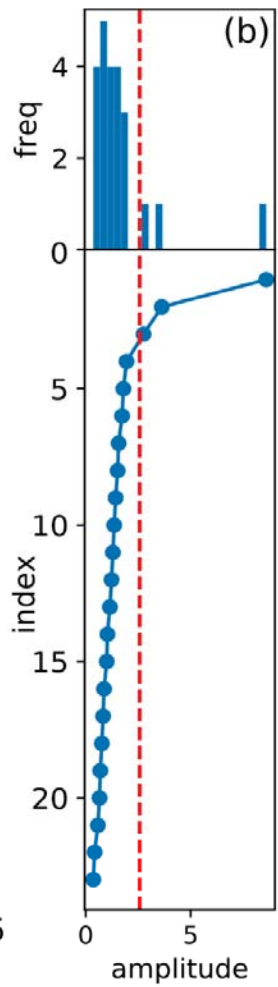
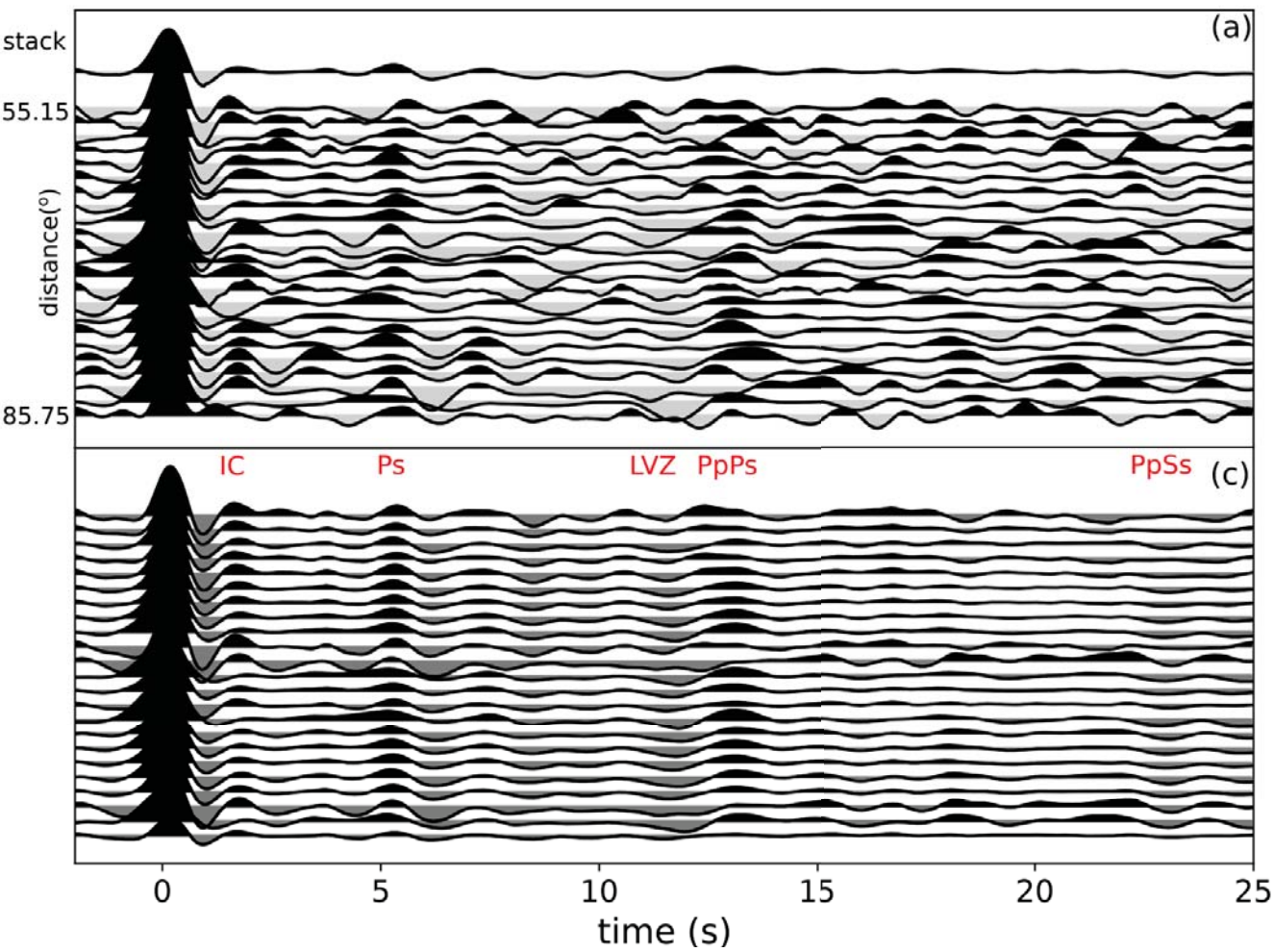


Figure 3.

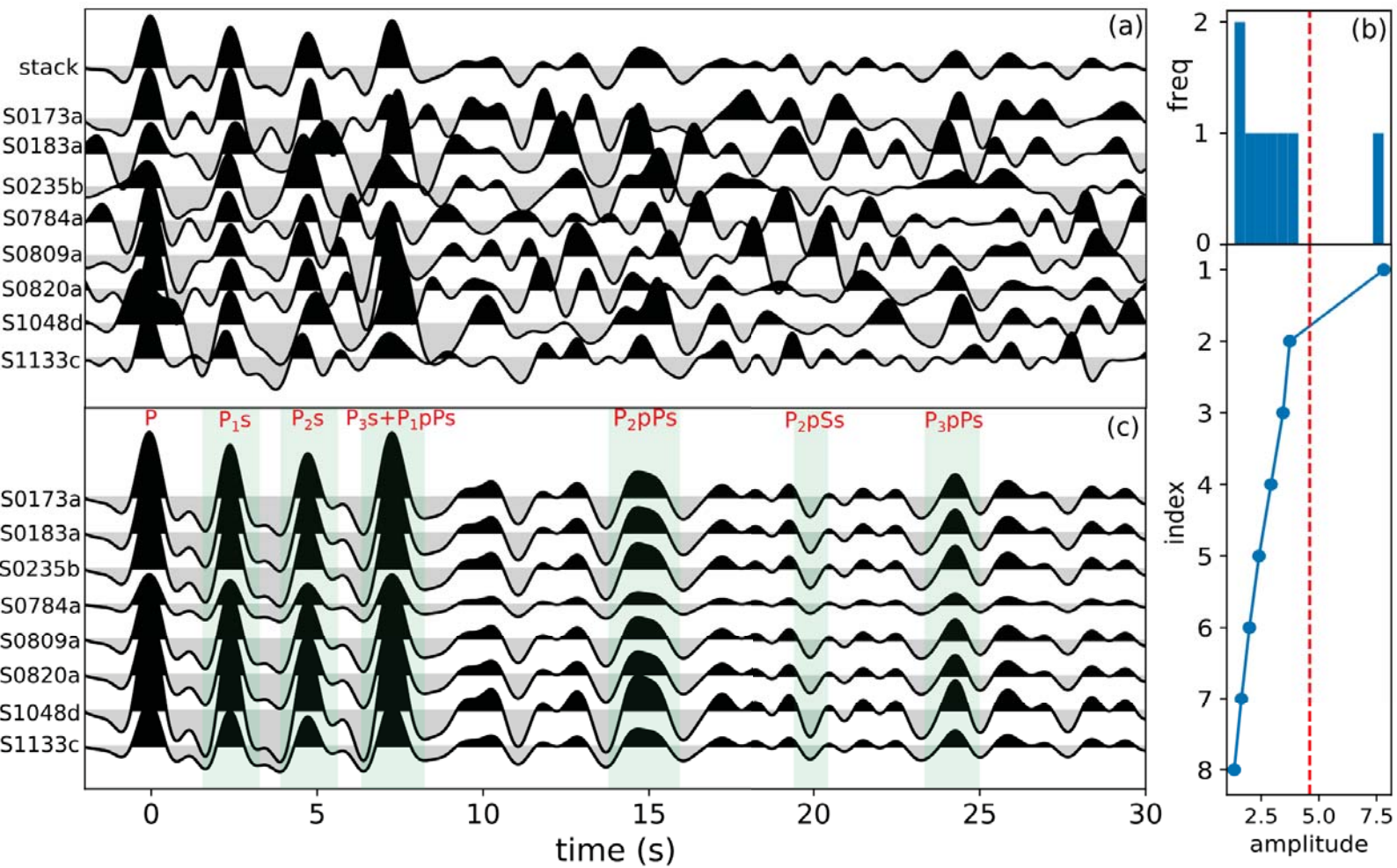


Figure 4.

